Engineering Notes

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Matrix Difference Equation Analysis of Vibrating Circumferentially Periodic Structures

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Introduction

REFERENCE 1 presents the matrix difference equation (MDE) method of calculating the free and forced vibrations of longitudinally periodic structures with simply supported and guided ends. Such structures are composed of identical substructures arrayed along a longitudinal axis. When the substructures are assumed to be symmetric about a median plane normal to the longitudinal axis, the analysis is simplified. The analysis of free undamped vibration is especially simple. All natural modes and frequencies of an extensive structure can be calculated by finding the eigensolutions of matrices of order equal to the number of degrees of freedom on one substructure boundary.

The purpose of this Note is to show the application of the method to circumferentially periodic or axisymmetric structures. Such a structure, shown in Fig. 1, is composed of identical substructures arrayed along a circular circumferential axis. The structure does not require regularity along the longitudinal axis, and any conditions of support can exist at the ends. The method of Ref. 1 is directly applicable to circumferentially periodic structures. No additional theoretical development is required.

Applications

In an application, one-half of the structure on one side of a longitudinal plane of symmetry is considered. For example, Fig. 2 shows a uniform circular ring. In Fig. 2a the ring is simply supported and in Fig. 2b it is guided, according to the definitions of Ref. 1. In a nonuniform structure, such as a stiffened shell, the condition of Fig. 1a yields vibration modes having nodes on the plane of symmetry, while the condition of Fig. 1b yields modes having zero tangential displacements and zero rotations on the plane of symmetry. In the case of the uniform circular ring, either condition provides all of the modes. Figure 3 shows a possible substructure—a straight beam segment with ends cut off so that the normal forces are tangential and the shear forces are radial. The angle $\theta = \pi/s$, where s is the number of substructures in the semicircumference. With boundary conditions and substructures chosen in this manner, the ring meets all of the conditions required by the analysis of Ref. 1. Table 1 shows computed radial natural frequencies compared with theoretical results for a ring having the following properties: radius = 12 in., cross-sectional area = 0.0036 in.^{2}, section moment of inertia = 0.108×10^{-5} in.^{4}, Young's modulus = 10.3×10^6 lb/in.^{2}, density = 0.000259 lb-s^2/in.^{4}, number of substructures = 18.

The natural modes and frequencies of a simply supported uniform circular cylindrical shell were also calculated. Figure 4a shows the shell, for which Poisson's ratio = 0.3, density = 0.000259 lb-s^2/in.^{4}, and Young's modulus = 10.3×10^6 lb/in.^{2}. Figure 4b shows a substructure, which has 142 degrees of freedom per boundary. Table 2 shows calculated natural frequencies compared with Flugge's theory,[2] for the case of two waves in the circumference. Figure 5 shows corresponding modes for three of the frequencies. Equally satisfactory results were found for the case of one wave and five waves in the circumference.

The method of forced vibration calculation of Ref. 1 is also expected to be applicable to circumferentially periodic structures.

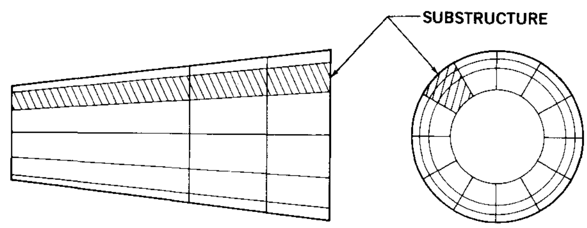

Fig. 1 Circumferentially periodic structure.

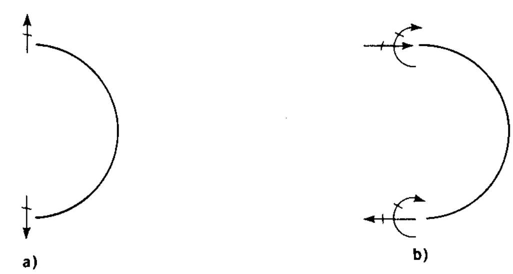

Fig. 2 Circular ring.

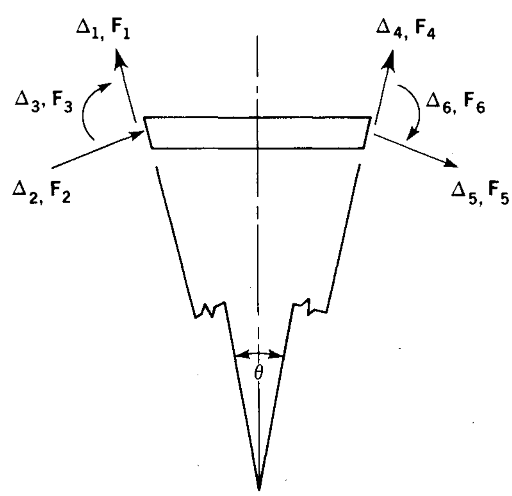

Fig. 3 Circular ring substructure.

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Table 1 Natural frequencies of a uniform circular ring

MDE, Hz	Classical, Hz	Percent error	MDE, HZ	Classical HZ	Percent error
0.00	0.00	a	c	376.06	c
0.00	0.00	b	c	456.23	c
10.28	10.24	0.31	c	544.03	c
29.01	28.97	0.14	c	639.46	c
55.53	55.55	−0.05	c	742.53	c
89.66	89.84	−0.21	c	853.24	c
131.44	131.80	−0.27	c	971.58	c
181.08	181.40	−0.18	c	1097.56	c
238.98	238.65	0.14	1238.87	1231.18	0.62
305.80	303.54	0.74			

[a] Rigid-body rotation. [b] Rigid-body translation. [c] Not calculated.

Table 2 Natural frequencies of the cylindrical shell, two waves in the circumference

m^a	1	3	3	4	5	6
$\Omega_{MDE}{}^b$	0.322	0.626	0.781	0.867	0.928	0.983
Ω_{Theo}	0.32	0.64	0.80	0.87	0.91	0.95
Percent difference	1	−2	−2	0	2	3

[a] m = number of half waves in length. [b] Ω = dimensionless frequency.

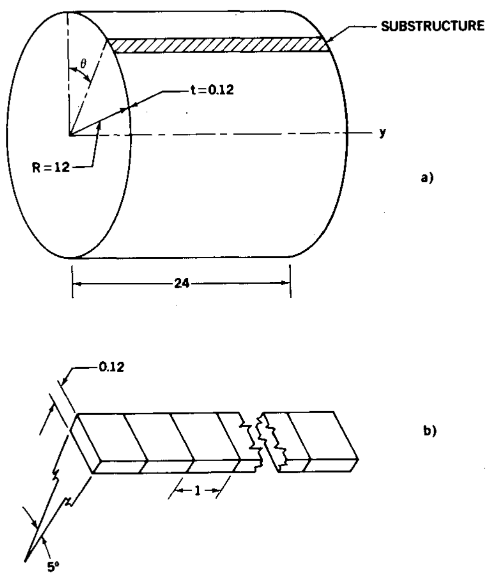

Fig. 4 Uniform circular cylindrical shell.

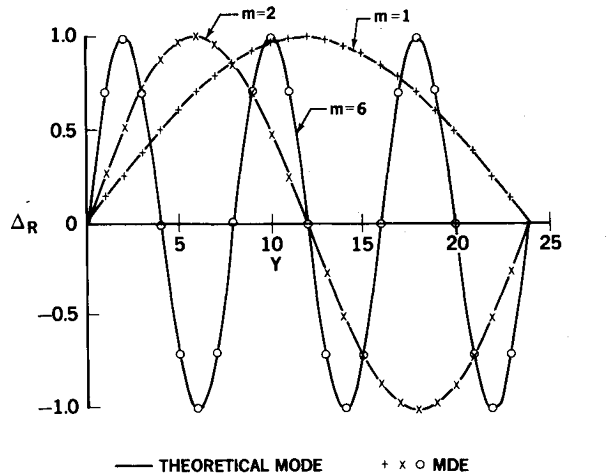

Fig. 5 Natural modes: radial displacement at $\theta = 0$, two waves in the circumference.

Acknowledgments

John Pickard of the Structural Mechanics Subdivision, Douglas Aircraft Company, contributed a key idea concerning the boundary conditions at the plane of symmetry of the circumferentially periodic structure.

References

[1] Denke, P.H., "Vibrating Spatially Periodic Structures with Simply Supported and Guided Ends," SAE Technical Paper 791064, Dec. 1979.

[2] Leissa, A.W., "Vibration of Shells," NASA SP-288, 1973.